

SPECIAL THEORY OF RELATIVITY

EQUIVALENCE OF MASS AND ENERGY

MASS-ENERGY RELATION

The mass 'm' of a body moving with velocity 'v' relative to a stationary observer varies with 'v' and is given by-

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1)$$

where m_0 is the mass of the object at rest, m is the mass of the object moving with velocity v , c is the velocity of light, v is the velocity of the object.

Squaring both sides of equation (1) we get,

$$m_0^2 = m^2 \left(1 - \frac{v^2}{c^2} \right)$$

$$\Rightarrow m^2 c^2 - m^2 v^2 = m_0^2 c^2$$

On differentiation we get,

$$\Rightarrow 2mc^2 dm - [2mv^2 dm + 2m^2 v dv] = 0$$

$$\Rightarrow c^2 dm = v^2 dm + mv dv \quad (2)$$

Now,

$$\text{Workdone} = dk = dw = F ds \quad (3)$$

From Newton's 2nd law of motion,

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) = m \frac{dv}{dt} + v \frac{dm}{dt}$$

From equation (3) we get,

$$dk = \left(m \frac{dv}{dt} + v \frac{dm}{dt} \right) ds$$

$$\Rightarrow dk = m \frac{ds}{dt} dv + v \frac{ds}{dt} dm$$

$$\Rightarrow dk = mv dv + v^2 dm \quad (4)$$

Comparing equations (2) and (4) we get,

$$dk = c^2 dm$$

Integrating we get,

$$\int_0^k dk = \int_{m_0}^m c^2 dm$$

$$\Rightarrow k = c^2(m - m_0)$$

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which is relativistic kinetic energy.

$$\text{Rest mass energy} = m_0 c^2$$

$$\text{Total Energy} = \text{Kinetic Energy} + \text{Rest Mass Energy}$$

$$= c^2(m - m_0) + m_0 c^2$$

$$= mc^2$$

$$\therefore E = mc^2$$