

SPECIAL THEORY OF RELATIVITY

VARIATION OF MASS WITH VELOCITY

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Mass is a function of the velocity of the body. It increases with increasing velocity represented by the relation

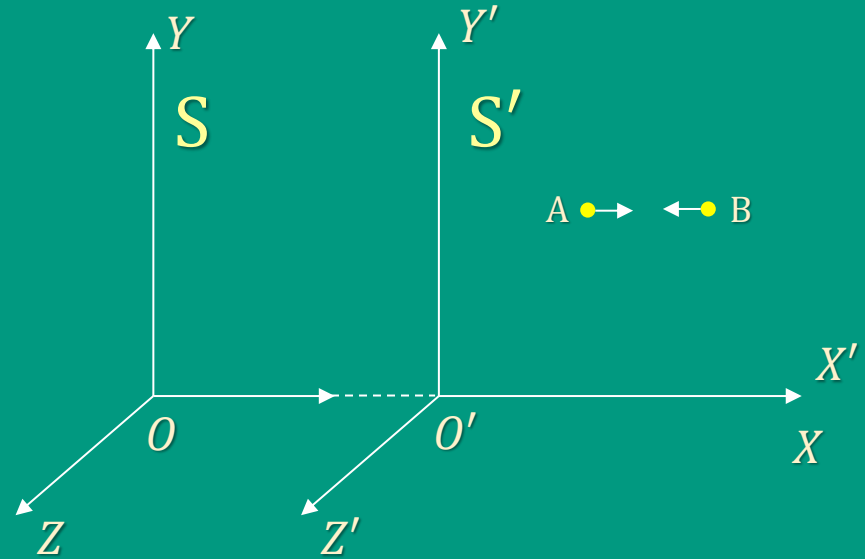
$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_0 is the mass of the object at rest, m is the mass of the object moving with velocity v , c is the velocity of light, v is the velocity of the object.

Let us consider two frame of references S and S' where S' is moving away from S with a constant velocity v relative to S .

Let u and $-u$ be the velocity of the balls A and B w.r.t. S' frame.

Let u_1 and u_2 be the velocity of the balls A and B w.r.t. frame of reference S .



$$u_1 = \frac{u + v}{1 + \frac{uv}{c^2}} \quad (1)$$

$$u_2 = \frac{-u + v}{1 - \frac{uv}{c^2}} \quad (2)$$

Applying law of conservation of momentum for frame of reference S .

Total initial momentum Total final momentum
before collision = after collision

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

$$\Rightarrow m_1 \left(\frac{u + v}{1 + \frac{uv}{c^2}} \right) + m_2 \left(\frac{-u + v}{1 - \frac{uv}{c^2}} \right) = (m_1 + m_2) v$$

$$\Rightarrow m_1 \left(\frac{u + v}{1 + \frac{uv}{c^2}} \right) - m_1 v = m_2 v - m_2 \left(\frac{-u + v}{1 - \frac{uv}{c^2}} \right)$$

$$\Rightarrow m_1 \left(\frac{u - u \frac{v^2}{c^2}}{1 + \frac{uv}{c^2}} \right) = m_2 \left(\frac{u - u \frac{v^2}{c^2}}{1 - \frac{uv}{c^2}} \right)$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1 + \frac{uv}{c^2}}{1 - \frac{uv}{c^2}} \quad (3)$$

$$\begin{aligned} (1)^2 &\Rightarrow u_1^2 = \left(\frac{u+v}{1 + \frac{uv}{c^2}} \right)^2 \\ &\Rightarrow 1 - \frac{u_1^2}{c^2} = 1 - \frac{1}{c^2} \left(\frac{u+v}{1 + \frac{uv}{c^2}} \right)^2 \\ &\Rightarrow 1 - \frac{u_1^2}{c^2} = \frac{\left(1 - \frac{u^2}{c^2} \right) \left(1 - \frac{v^2}{c^2} \right)}{\left(1 + \frac{uv}{c^2} \right)^2} \quad (4) \end{aligned}$$

Similarly,

$$1 - \frac{u_2^2}{c^2} = \frac{\left(1 - \frac{u^2}{c^2} \right) \left(1 - \frac{v^2}{c^2} \right)}{\left(1 - \frac{uv}{c^2} \right)^2} \quad (5)$$

Dividing (5) by (4),

$$\frac{1 - \frac{u_2^2}{c^2}}{1 - \frac{u_1^2}{c^2}} = \frac{\left(1 + \frac{uv}{c^2}\right)^2}{\left(1 - \frac{uv}{c^2}\right)^2}$$

$$\Rightarrow \frac{1 + \frac{uv}{c^2}}{1 - \frac{uv}{c^2}} = \frac{\sqrt{1 - \frac{u_2^2}{c^2}}}{\sqrt{1 - \frac{u_1^2}{c^2}}}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{\sqrt{1 - \frac{u_2^2}{c^2}}}{\sqrt{1 - \frac{u_1^2}{c^2}}}$$

$$\Rightarrow m_1 \sqrt{1 - \frac{u_1^2}{c^2}} = m_2 \sqrt{1 - \frac{u_2^2}{c^2}} = m_0 \quad (\text{any constant})$$

$$\Rightarrow m_1 \sqrt{1 - \frac{u_1^2}{c^2}} = m_0$$

$$\Rightarrow m_1 = \frac{m_0}{\sqrt{1 - \frac{u_1^2}{c^2}}}$$

Let, $m_1 = m$, $u_1 = v$

$$\therefore m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \blacksquare$$