LAWS OF ELECTROMAGNETISM IN GALILEAN TRANSFORMATION

The following are the four laws of electromagnetism, called *Maxwell's electromagnetic equations*, in empty space

1. Gauss's law in electricity: The net electric flux (i.e. electric lines of force) issuing from an enclosed empty space is zero. Mathematically,

$$div\vec{E} = 0 => \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \qquad (1)$$

where E_x, E_y and E_z are the components of $\vec{E}$ along the three spatial axes.

2. Gauss's law in magnetism: The net magnetic flux (i.e. magnetic lines of force) issuing from an enclosed empty space is zero. Mathematically,

$$div\vec{B} = 0 => \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \qquad (2)$$

where B_x, B_y and B_z are the components of $\vec{B}$ along the three spatial axes.

3. Faraday's law of electromagnetic induction: The net rate of change of magnetic flux through a loop produces an electric field along the loop. Mathematically,

$$\left.\begin{aligned}
curl\vec{E} &= -\frac{\partial B}{\partial t} \\
\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -\frac{\partial B_x}{\partial t} \\
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -\frac{\partial B_y}{\partial t} \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -\frac{\partial B_z}{\partial t}
\end{aligned}\right\} \tag{3}$$

4. Maxwell's law of electromagnetic induction: The net rate of change of electric flux through a loop produces a magnetic field along the loop. Mathematically,

$$\left.\begin{aligned}
curl\vec{B} &= \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\\
\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z} &= \mu_0\varepsilon_0\frac{\partial E_x}{\partial t}\\
\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x} &= \mu_0\varepsilon_0\frac{\partial E_y}{\partial t}\\
\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y} &= \mu_0\varepsilon_0\frac{\partial E_z}{\partial t}
\end{aligned}\right\} \tag{4}$$

where μ_0 and ε_0 are two constants called the permeability and permittivity respectively of empty space.

Let us assume that Maxwell's electromagnetic equations are *invariant under Galilean transformation*.

$$\frac{\partial E'_x}{\partial x'} + \frac{\partial E'_y}{\partial y'} + \frac{\partial E'_z}{\partial z'} = 0 \qquad \frac{\partial B'_x}{\partial x'} + \frac{\partial B'_y}{\partial y'} + \frac{\partial B'_z}{\partial z'} = 0 \tag{5}$$

$$\left.\begin{aligned} \frac{\partial E'_z}{\partial y'} - \frac{\partial E'_y}{\partial z'} &= -\frac{\partial B'_x}{\partial t'} \\ \frac{\partial E'_z}{\partial z'} - \frac{\partial E'_z}{\partial x'} &= -\frac{\partial B'_y}{\partial t'} \\ \frac{\partial E'_z}{\partial x'} - \frac{\partial E'_z}{\partial y'} &= -\frac{\partial B'_z}{\partial t'} \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned}
\frac{\partial B'_z}{\partial y'} - \frac{\partial B'_y}{\partial z'} &= \mu_0 \varepsilon_0 \frac{\partial E'_x}{\partial t'} \\
\frac{\partial B'_x}{\partial z'} - \frac{\partial B'_z}{\partial x'} &= \mu_0 \varepsilon_0 \frac{\partial E'_y}{\partial t'} \\
\frac{\partial B'_y}{\partial x'} - \frac{\partial B'_x}{\partial y'} &= \mu_0 \varepsilon_0 \frac{\partial E'_z}{\partial t'}
\end{aligned}\right\} \tag{7}$$

Now,

$$\frac{\partial}{\partial x} = \frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} + \frac{\partial y'}{\partial x}\frac{\partial}{\partial y'} + \frac{\partial z'}{\partial x}\frac{\partial}{\partial z'} + \frac{\partial t'}{\partial x}\frac{\partial}{\partial t'}$$

$$= \frac{\partial (x - vt)}{\partial x}\frac{\partial}{\partial x'} + 0 + 0 + 0 = \frac{\partial}{\partial x'} \tag{8}$$

Similarly,

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y'} \qquad\qquad \frac{\partial}{\partial z} = \frac{\partial}{\partial z'} \tag{9}$$

$$\frac{\partial}{\partial t} = \frac{\partial x'}{\partial t}\frac{\partial}{\partial x'} + \frac{\partial y'}{\partial t}\frac{\partial}{\partial y'} + \frac{\partial z'}{\partial t}\frac{\partial}{\partial z'} + \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'}$$

$$= \frac{\partial(x - vt)}{\partial t}\frac{\partial}{\partial x'} + 0 + 0 + \frac{\partial}{\partial t'}$$

$$= \frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'} \tag{10}$$

Using equations (8) and (9) in (1) we get,

$$\frac{\partial E_x}{\partial x'} + \frac{\partial E_y}{\partial y'} + \frac{\partial E_z}{\partial z'} = 0 \tag{11}$$

Comparing equations (5) and (11) we get,

$$E'_x = E_x, \qquad E'_y = E_y, \qquad E'_z = E_z, \tag{12}$$

Using equations (8) and (9) in (2) we get,

$$\frac{\partial B_x}{\partial x'} + \frac{\partial B_y}{\partial y'} + \frac{\partial B_z}{\partial z'} = 0 \tag{13}$$

Comparing equations (5) and (13) we get,

$$B'_x = B_x, \qquad B'_y = B_y, \qquad B'_z = B_z, \tag{14}$$

Using equations (9) and (10) in first equation of (3) we get,

$$\frac{\partial E_z}{\partial y'} - \frac{\partial E_y}{\partial z'} = -\left[\frac{\partial B_x}{\partial t'} - v\frac{\partial B_x}{\partial x'}\right]$$

$$=> \frac{\partial E'_z}{\partial y'} - \frac{\partial E'_y}{\partial z'} = -\left[\frac{\partial B_x}{\partial t'} - v\frac{\partial B_x}{\partial x'}\right] \qquad (16)$$

Comparing equations (6) and (16) we get,

$$\frac{\partial B'_x}{\partial t'} = \frac{\partial B_x}{\partial t'} - v\frac{\partial B_x}{\partial x'}$$

But according to equation (14) we should not have the second term, which is a contradiction. Hence our assumption is wrong.

Therefore, the laws of electromagnetism are *not invariant* under the Galilean transformation. In other words, classical relativity applies only to classical mechanics and not to electromagnetism.

- The laws of mechanics are invariant under Galilean transformations, whereas *electrodynamics and Maxwell's equations* are varying under the above transformations.
- This shows that the velocity of light will have different values for different observers moving with different uniform velocities.
- But the *speed of light is invariant* in all inertial frames of reference, hence, Galilean transformations need to be modified.
- Galilean transformations do not predict accurate results when bodies move with *speeds closer to the speed of light*.
- Hence, Lorentz transformations are used when bodies travel at such speeds.
- Lorentz deduced the transformation equations, which are in agreement with the results of *Michelson–Morley experiment*.