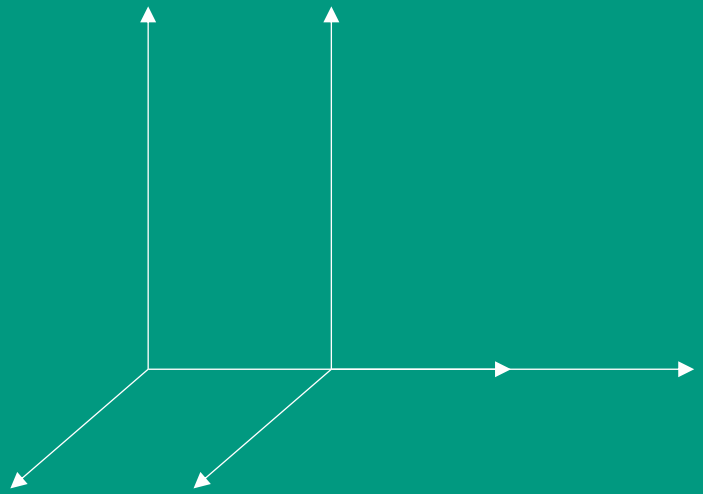
SPECIAL THEORY OF RELATIVITY

MATSPL25024B



Dr. Rajshekhar Roy Baruah

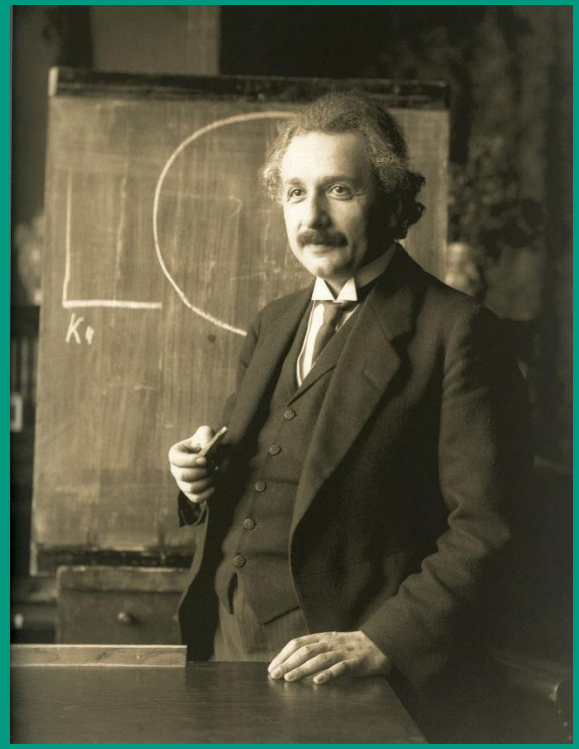
Special Theory of Relativity

Space and Time

(1905)

General Theory of Relativity

(1915)



"Einstein's theory of relativity may be regarded as based on the fundamental idea of relativity of all motion."

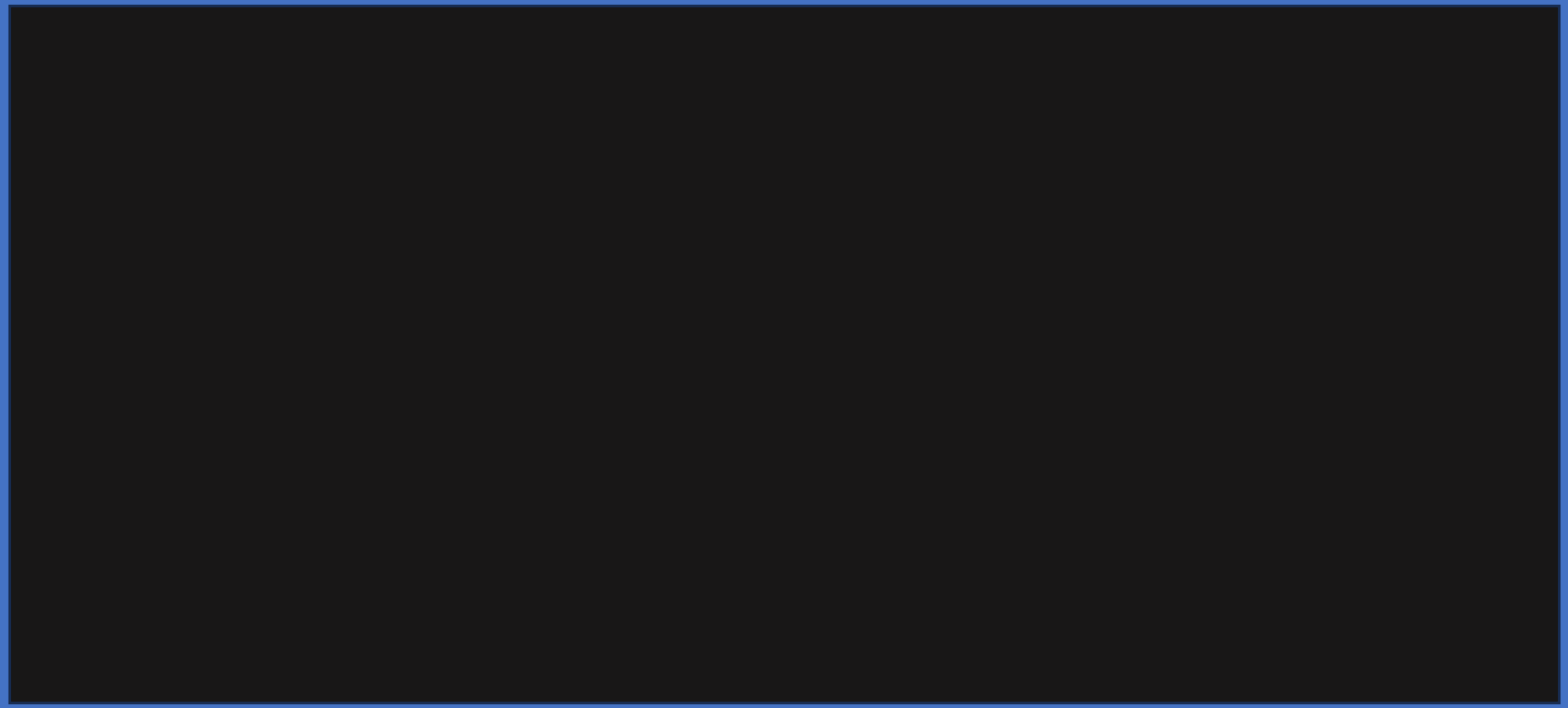
R. C. Tolman

We know that *all motions* are *relative*, in other words, the motion of a body is detected only relative to or with respect to some other body. Therefore, there is nothing like *absolute motion* i.e. *no motion is detectable* without reference to a second body.

Motion and Relative motion:

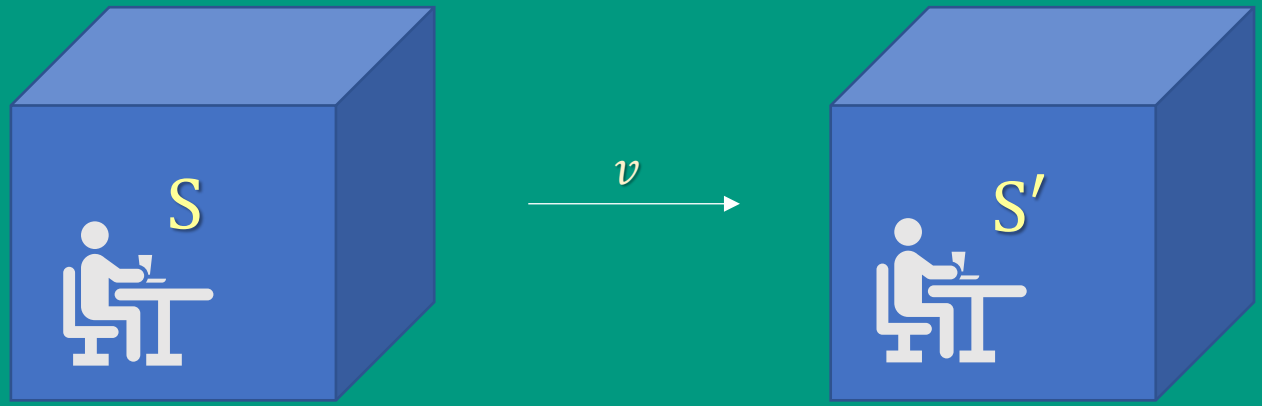
Motion refers to the *change in the position* of an object over time. It is a fundamental concept in physics and plays a crucial role in understanding the behavior of the physical world.

Relative motion comes into play when considering the motion of *one object with respect to another*. It is the concept of observing the motion of an object from the perspective of another object that is also in motion.



There are *two* types of relative motion:

1. Uniform relative motion (e.g. a train moving in a certain direction with a certain speed relative to trees, houses, etc.) (Special theory of relativity)
2. Non-uniform relative motion (e.g. a train starting from rest.) (General theory of relativity)



Will the two scientists discover the same laws of physics?

All laws of physics must be the same in all laboratories in uniform relative motion.

This is called the *principle of relativity*.

Laws of physics ≡ Mathematical Equations

In classical mechanics or Newtonian mechanics any kind of motion is meaningful only when it is referred to some fixed frame of reference.

The motion is described by

$$\frac{d}{dt}(mv) = F$$

In absence of external force

$$\frac{d}{dt}(mv) = 0$$

$$v = constant$$

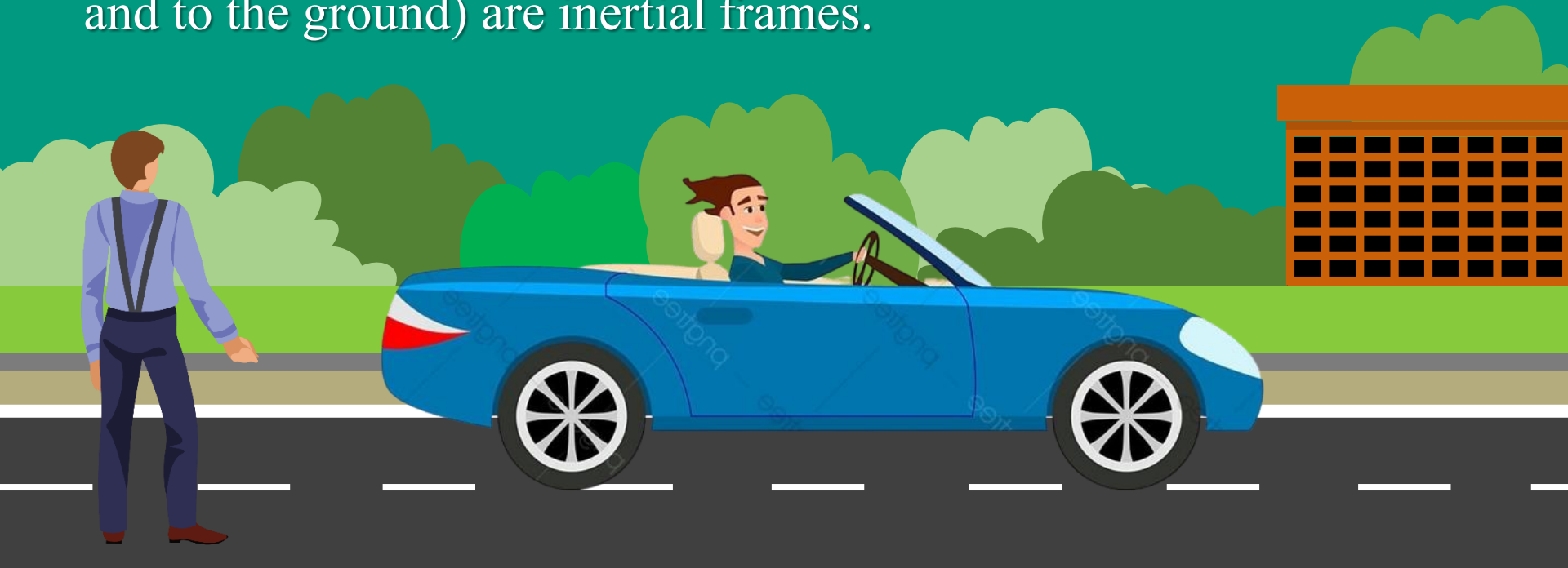
FRAME OF REFERENCE

A frame of reference is a *set of coordinates* (for example x, y & z axes) with respect to whom any physical quantity can be determined.

INERTIAL FRAME OF REFERENCE

An *inertial frame of reference* is a frame of reference in which the law of inertia and other physics laws are valid. Any frame moving at a constant velocity relative to another frame is also an inertial frame of reference.

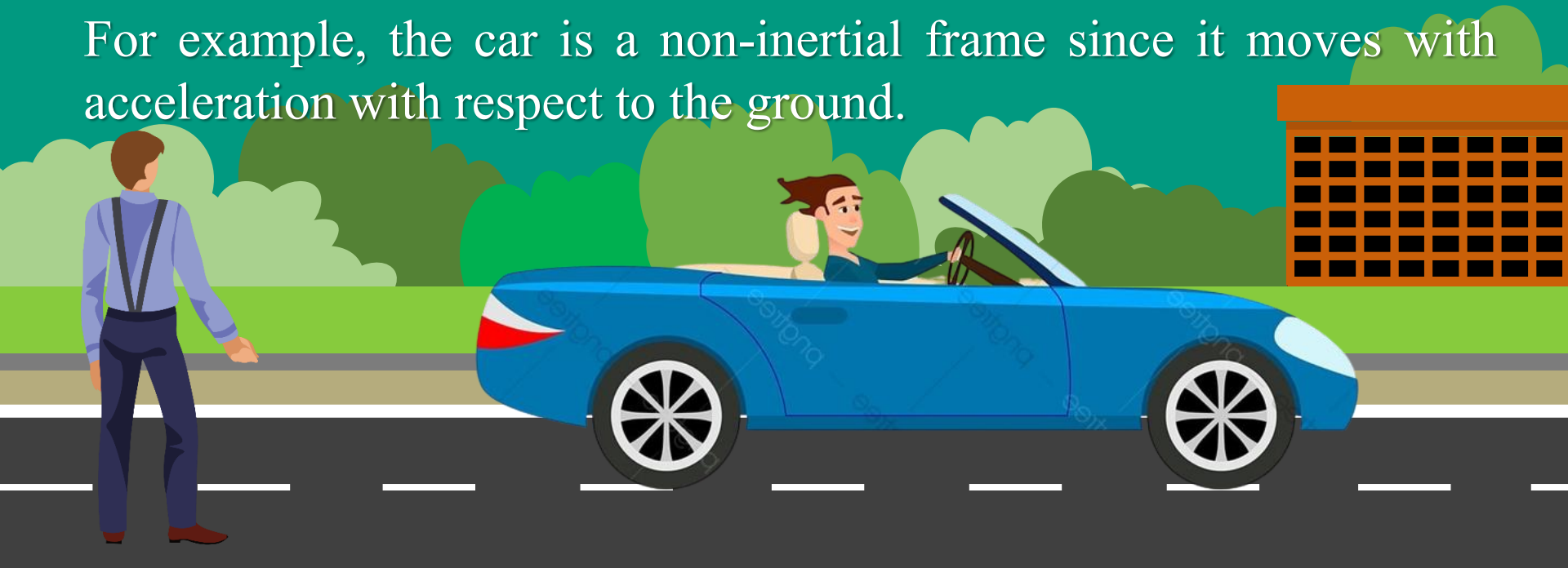
For example, the car is moving with *uniform velocity* $\vec{v}$ with respect to a person standing (*at rest*) on the ground. As the car is moving with constant velocity with respect to the ground to the person is at rest on the ground, *both frames* (with respect to the car and to the ground) are inertial frames.



NON-INERTIAL FRAME OF REFERENCE

A *non-inertial frame of reference* is a reference frame in which the law of inertia does not hold.

For example, the car is a non-inertial frame since it moves with acceleration with respect to the ground.



Newtonian Mechanics:

Classical mechanics is often called Newtonian mechanics because nearly the entire study builds on the work of Isaac Newton.

Newtonian Mechanics is based on application of Newton's laws of motion which assume that the concept of distance, time, and mass are absolute, i.e., motion is in an *inertial frame*.

Fundamental *postulates* of special theory of relativity

- The laws of physics are the *same* in all inertial reference frames.
- The speed of light in a *vacuum* is *constant* and is the same for all observers, regardless of their relative motion.

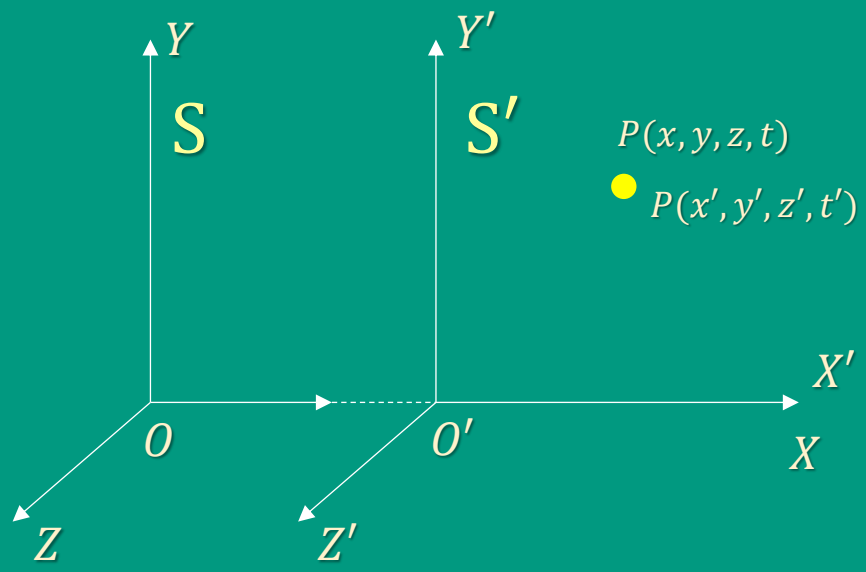
GALILEAN TRANSFORMATION

The transformation of coordinates of a particle from one *inertial frame* of reference to another *inertial frame* of reference is called *Galilean transformation*.

Let us consider two inertial frame of references S and S' having cartesian coordinate axes as XYZ and $X'Y'Z'$ and origins O and O' respectively.

At time $t = 0$, both the frames are at rest so that their origin O and O' coincide with each other.

Now let, S' frame start moving away from S with constant velocity v along the positive direction of *x-axis*.



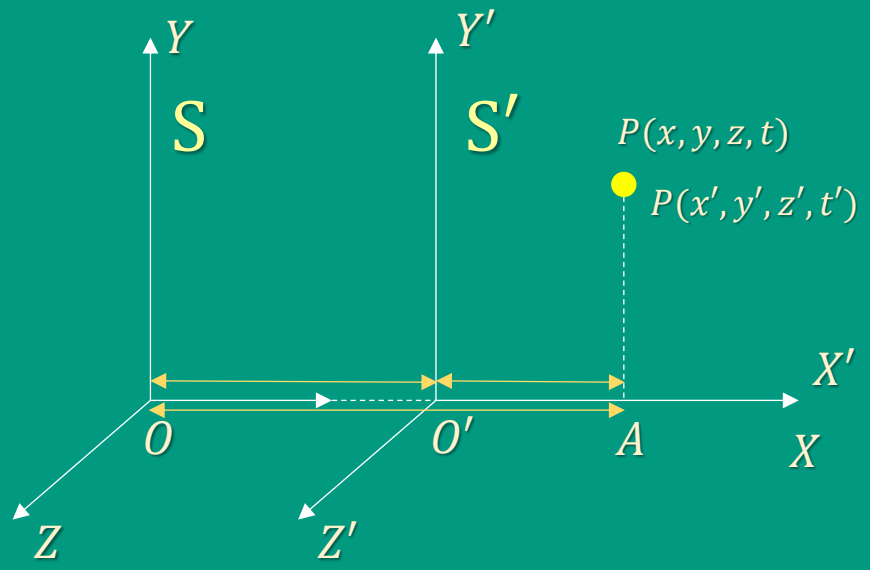
Let an event occurs at point P at any instant of time. The coordinates of point P w.r.t. observer O in S frame are (x, y, z, t).

The coordinates of the point P w.r.t. the observer O′ in S' frame are (x', y', z', t').

From the figure we have,

$$OA = OO' + O'A$$

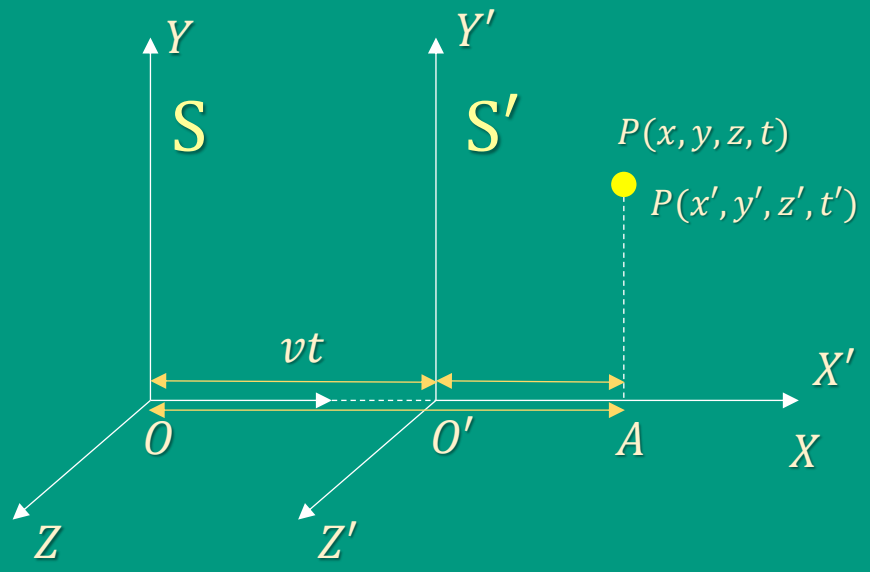
$$=> x = vt + x' \;\; => x' = x - vt$$



As there is no relative motion along y and z axis therefore

$$y = y', \quad z = z'$$

As the time is considered to be absolute in nature i.e. time remains same in all inertial frames of references so $t = t'$.



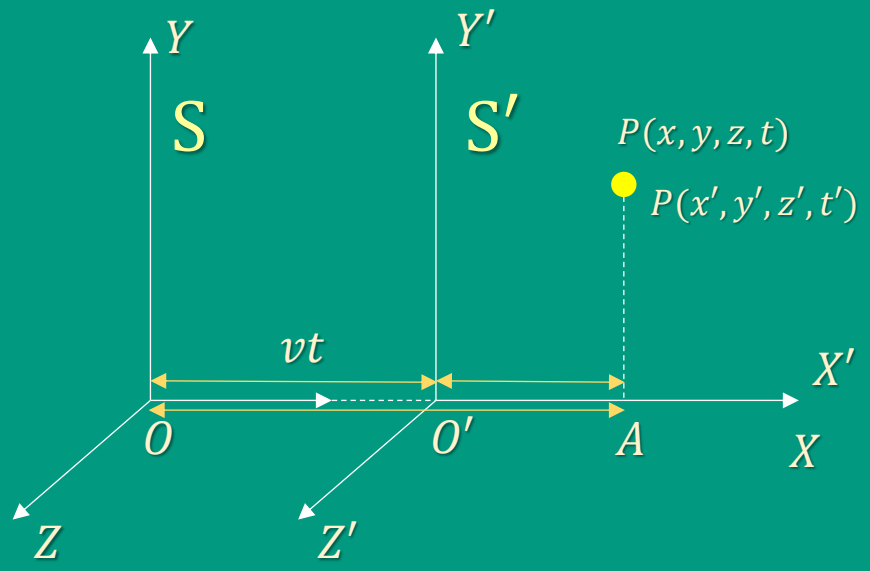
∴ The required Galilean transformation equations are

$$x' = x - vt, \quad y = y', \quad z = z', \quad t = t'$$

Velocity Transformation:

Let the velocity of P be V according to S at time t and V′ according to S' at the corresponding time t'.

Let P be moving in the x-direction under the action of a force.

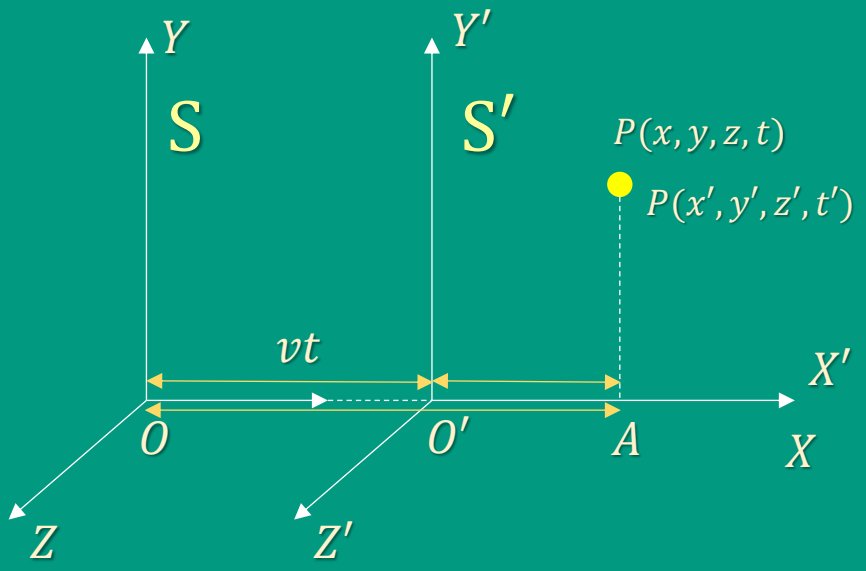


According to S	According to S'
Mass $= m$	Mass $= m'$
Force $= F$	Force $= F'$
Position coordinates of P (x,y,z,t) at t	Position coordinates of P (x′,y′,z′,t′) at t′

$$V' = \frac{dx'}{dt'} = \frac{d(x - vt)}{dt}$$

$$= \frac{dx}{dt} - v$$

$$= V - v$$



Acceleration Transformation:

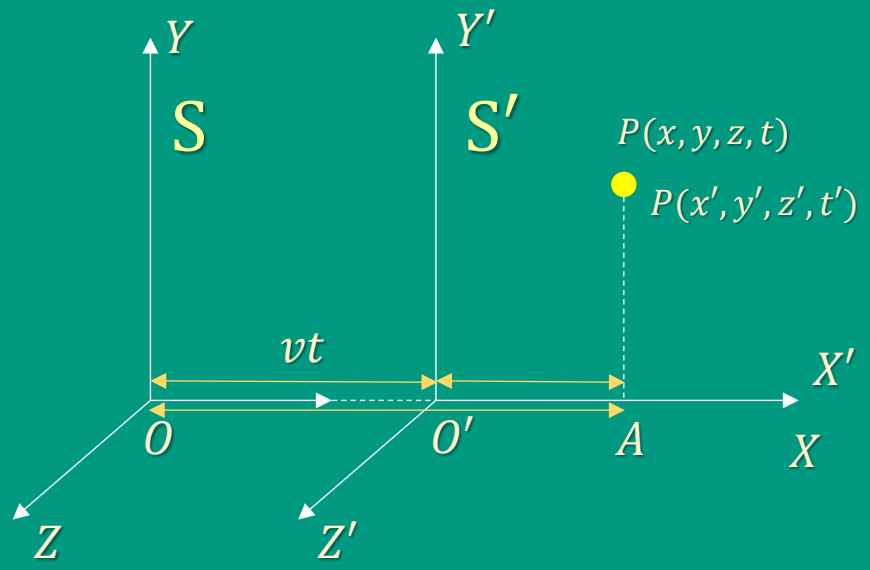
Let the acceleration of P be a according to S at time t and a' according to S' at the corresponding time t'.

$$a' = \frac{d}{dt'}\left(\frac{dx'}{dt'}\right) = \frac{d}{dt}\left[\frac{d(x-vt)}{dt}\right]$$

$$= \frac{d}{dt}\left(\frac{dx}{dt} - v\right)$$

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right) - 0$$

$$= a$$



In classical physics, $m = m'$ and $F = F'$

Let us suppose that Newton's law of motion is found to be valid in S, i.e. $F = ma$.

$$\therefore F' = m'a'$$

So, if Newton's law is valid in S, then it is also valid in S'.

Since Newton's laws of motion are the basis of all laws of classical mechanics, we arrive at the following principle of relativity-

All laws of classical mechanics are valid in any two co-ordinate systems moving uniformly relative to each other.

This is called the principle of classical mechanics or Newtonian relativity.

Thank You