

Partial Differential Equations
MATADL14034

Q1. A partial differential equation involves:

- (A) Ordinary derivatives only
- (B) Partial derivatives only
- (C) Both ordinary and partial derivatives
- (D) None of the above

Answer: (B)

Q2. The order of the PDE

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^3 z}{\partial y^3} = 0$$

is:

- (A) 2
- (B) 3
- (C) 1
- (D) 0

Answer: (B)

Q3. The PDE

represents a:

- (A) Linear equation
- (B) Nonlinear equation
- (C) Homogeneous equation
- (D) Compatible equation

Answer: (B)

Q4. If $z = f(x + y)$, then

$$\frac{\partial z}{\partial x} =$$

- (A) $f'(x + y)$
- (B) $f'(x - y)$
- (C) $f'(x + y) + f'(x - y)$
- (D) 0

Answer: (A)

Q5. A first order PDE can be expressed as:

$$F(x, y, z, p, q) = 0,$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. This is called:

- (A) Lagrange's linear equation
- (B) General first order PDE
- (C) Jacobi's form
- (D) Canonical form

Answer: (B)

Q6. The Cauchy problem for first order equations refers to:

- (A) Finding a general solution
- (B) Solving with given initial conditions
- (C) Eliminating arbitrary constants
- (D) None of these

Answer: (B)

Q7. In Cauchy's problem, the data are given along a:

- (A) Point
- (B) Curve
- (C) Plane
- (D) Surface

Answer: (B)

Q8. The general integral of a first order PDE contains:

- (A) One arbitrary constant
- (B) Two arbitrary constants
- (C) One arbitrary function
- (D) Two arbitrary functions

Answer: (C)

Q9. The complete integral of a PDE is the solution containing:

- (A) No arbitrary constant
- (B) One arbitrary function
- (C) Two arbitrary constants
- (D) Three arbitrary constants

Answer: (C)

Q10. The equation $p + q = 1$ is:

- (A) Linear and homogeneous
- (B) Linear and non-homogeneous
- (C) Nonlinear
- (D) None of these

Answer: (B)

Q11. The linear first order PDE of Lagrange's type is:

$$Pp + Qq = R$$

where P, Q, R are functions of:

- (A) x, y, z
- (B) p, q
- (C) x, y
- (D) x, y, z, p, q

Answer: (A)

Q12. The method used to solve $Pp + Qq = R$ is:

- (A) Cauchy's method of characteristics
- (B) Separation of variables
- (C) Integration factor method
- (D) None of these

Answer: (A)

Q13. The subsidiary equations corresponding to $Pp + Qq = R$ are:

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

These are called:

- (A) Characteristic equations
- (B) Auxiliary equations
- (C) Integrating equations
- (D) Ordinary equations

Answer: (A)

Q14. The compatible system of first order equations means:

- (A) The equations have no solution
- (B) The equations have at least one common solution
- (C) The equations are linearly independent
- (D) The equations are homogeneous

Answer: (B)

Q15. The characteristic curves are defined as:

- (A) Curves along which the PDE becomes ODE
- (B) Curves of constant z
- (C) Orthogonal trajectories
- (D) None of these

Answer: (A)

Q16. The PDE $p^2 + q^2 = 1$ represents:

- (A) Plane
- (B) Cone
- (C) Cylinder
- (D) Sphere

Answer: (D)

Q17. If $z = xy + f(x + y)$, then the PDE is:

- (A) $p + q = 1$
- (B) $p + q = 0$
- (C) $p + q = 2$
- (D) $p - q = 0$

Answer: (A)

Q18. The equation $z = f(x^2 + y^2)$ leads to:

$$p = \frac{\partial z}{\partial x} = 2xf'(x^2 + y^2)$$

Hence,

- (A) $p^2 + q^2 = 4(x^2 + y^2)[f'(x^2 + y^2)]^2$
- (B) $p^2 + q^2 = 1$
- (C) $p^2 - q^2 = 0$
- (D) $pq = 0$

Answer: (A)

Q19. The PDE whose complete integral is $z = ax + by + c$ is:

- (A) $p = a, q = b$
- (B) $\frac{\partial^2 z}{\partial x^2} = 0$
- (C) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$
- (D) None of these

Answer: (B)

Q20. The function $z = f(x + y) + g(x - y)$ satisfies:

- (A) Laplace's equation
- (B) Wave equation
- (C) Heat equation
- (D) Poisson's equation

Answer: (B)

Q21. A PDE is said to be nonlinear if:

- (A) z appears linearly
- (B) Highest derivative appears nonlinearly
- (C) Coefficients are constants
- (D) Order is one

Answer: (B)

Q22. The number of arbitrary constants in the complete integral of a first order PDE is:

- (A) 1
- (B) 2
- (C) 3
- (D) Infinite

Answer: (B)

Q23. The equation $p^2 + q^2 = z^2$ is:

- (A) Linear
- (B) Nonlinear
- (C) Homogeneous
- (D) Linear and homogeneous

Answer: (B)

Q24. The Cauchy's method of characteristics converts a PDE into:

- (A) Integral equation
- (B) Algebraic equation
- (C) System of ODEs
- (D) None of these

Answer: (C)

Q25. Jacobi's method is used to find:

- (A) Singular solution
- (B) Complete integral
- (C) General solution
- (D) Particular integral

Answer: (A)

Q26. The singular solution is obtained by:

- (A) Eliminating arbitrary constants
- (B) Eliminating arbitrary functions
- (C) Differentiating with respect to constants and eliminating them
- (D) None of these

Answer: (C)

Q27. The envelope of the family of surfaces represented by a complete integral gives:

- (A) General solution
- (B) Singular solution
- (C) Particular solution
- (D) Complementary solution

Answer: (B)

Q28. The compatibility condition for two first order PDEs ensures:

- (A) A unique common solution exists
- (B) No solution exists
- (C) Infinite solutions exist
- (D) The PDEs are linear

Answer: (A)

Q29. The auxiliary equations of Lagrange's equation are integrated to find:

- (A) Two independent integrals
- (B) One integral only
- (C) Particular integral
- (D) None of these

Answer: (A)

Q30. The general solution of $Pp + Qq = R$ is of the form:

- (A) $f(u, v) = 0$
- (B) $z = f(x, y)$
- (C) $u + v = 0$
- (D) $f(x, y, z) = 0$

Answer: (A)

Q31. The first order PDE which contains only one independent variable after using characteristics is called:

- (A) Compatible system
- (B) Reduced equation
- (C) Complete integral
- (D) Characteristic ODE

Answer: (D)

Q32. The order of a partial differential equation is determined by

- A) Highest power of derivative
- B) Highest order derivative present
- C) Highest power of the dependent variable
- D) Highest power of the independent variable

Answer: (B)

Q33. A compatible system of first-order partial differential equations implies

- A) Equations are inconsistent
- B) Equations are linearly independent
- C) Equations are linearly dependent
- D) A common solution exists

Answer: (D)

Q34. In Jacobi's method, the complete integral of a first-order partial differential equation contains

- A) One arbitrary constant
- B) Two arbitrary constants
- C) As many constants as independent variables
- D) Only one arbitrary function

Answer: (C)

Q35. A Clairaut's form of a first-order partial differential equation is

- A) $z = px + qy + f(p, q)$
- B) $z = f(x, y)$
- C) $z = p + q$
- D) $z^2 = p^2 + q^2$

Answer: (A)

Q36. The partial differential equation $u_{xx} + u_{yy} = 0$ is classified as

- A) Elliptic
- B) Hyperbolic
- C) Parabolic
- D) Circle

Answer: (A)

Q37. The vibrating membrane problem is governed by

- A) Laplace equation
- B) Wave equation in two dimensions

- C) Diffusion equation
- D) Poisson's equation

Answer: (B)

Q38. The order of a partial differential equation is determined by

- A) The number of independent variables
- B) The highest order derivative present
- C) The degree of the dependent variable
- D) The number of terms

Answer: (B)

Q39. The general form of a second-order PDE in two variables is

$$A \frac{\partial^2 z}{\partial x^2} + B \frac{\partial^2 z}{\partial x \partial y} + C \frac{\partial^2 z}{\partial y^2} + D \frac{\partial z}{\partial x} + E \frac{\partial z}{\partial y} + Fz = G,$$

where A, B, C, D, E, F, G are functions of

- A) x, y
- B) x, y, z
- C) x, y, z, p, q
- D) Only z

Answer: (A)

Q40. The equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

represents:

- (A) Laplace's equation
- (B) Heat equation
- (C) Wave equation
- (D) Poisson's equation

Answer: (C)

Q41. The Laplace equation $u_{xx} + u_{yy} = 0$ is classified as

- (A) Parabolic
- (B) Hyperbolic
- (C) Elliptic
- (D) Nonlinear

Answer: (C)

Q42. The classification of a second-order PDE $Au_{xx} + Bu_{xy} + Cu_{yy} = 0$ depends on

- (A) $A + B + C$
- (B) $A^2 + B^2 + C^2$
- (C) Discriminant $B^2 - 4AC$
- (D) $A + C$

Answer: (C)

Q43. For a linear PDE with constant coefficients, the coefficients of derivatives are

- (A) Functions of x, y

- (B) Constants
- (C) Functions of z
- (D) Zero

Answer: (B)

Q44. In the method of separation of variables, the solution is assumed as

- (A) Sum of functions of different variables
- (B) Product of functions of different variables
- (C) Difference of functions
- (D) Exponential of functions

Answer: (B)

Q45. The solution of Laplace's equation by separation of variables leads to

- (A) Two ordinary differential equations
- (B) A nonlinear equation
- (C) An integral equation
- (D) A higher-order PDE

Answer: (A)

Q46. The Fourier transform method is mainly applied to

- (A) Algebraic equations
- (B) Linear PDEs with constant coefficients
- (C) Nonlinear equations
- (D) Integral equations only

Answer: (B)

Q47. The Laplace transform method is best suited for solving

- (A) Linear PDEs with initial value problems
- (B) Nonlinear PDEs
- (C) Elliptic equations
- (D) Periodic boundary problems

Answer: (A)

Q48. A nonlinear PDE is one in which

- (A) The dependent variable appears linearly
- (B) The coefficients of derivatives are constants
- (C) The derivatives appear with powers or products
- (D) The order is higher than one

Answer: (C)

Q49. The one-dimensional heat equation is

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}.$$

Here k represents:

- (A) Density
- (B) Specific heat
- (C) Thermal diffusivity

(D) Temperature

Answer: (C)

Q50. The general integral of a homogeneous linear PDE with constant coefficients is obtained using

(A) Direct integration

(B) Auxiliary equation and complementary function

(C) Method of separation of variables

(D) Fourier expansion

Answer: (B)

