1. **Define a partial differential equation.**

   A **partial differential equation (PDE)** is an equation involving partial derivatives of an unknown function of two or more independent variables.
   Example: $u_{xx} + u_{yy} = 0$ is a PDE in x and y.

2. **What is meant by the order of a PDE?**

   The **order** of a PDE is the order of the highest derivative present in the equation.

3. **Write the standard form of a first-order PDE.**

   A first-order PDE can be written as

   $$F(x, y, z, p, q) = 0,$$

   where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

4. **What is meant by a complete integral of a first-order PDE?**

   A **complete integral** is a solution of the form

   $$z = f(x, y, a, b),$$

   containing as many arbitrary constants as there are independent variables (usually two).

5. **What is the origin of a first-order PDE?**

   First-order PDEs often arise by eliminating arbitrary constants or functions from a relation involving x, y, z.

6. **State Cauchy's problem for a first-order PDE.**

   Cauchy's problem consists of finding a solution $z(x, y)$ satisfying a given first-order PDE and passing through a prescribed initial curve (or surface).

7. **Write the general linear first-order PDE.**

   The general form is

   $$a(x, y, z)p + b(x, y, z)q = c(x, y, z),$$

   where $p = z_x$, $q = z_y$.

8. **What are the subsidiary equations in Cauchy's method of characteristics?**

   The subsidiary (characteristic) equations are

   $$\frac{dx}{a} = \frac{dy}{b} = \frac{dz}{c}.$$

9. **What is the geometrical meaning of characteristic curves?**

   Characteristic curves are the curves along which the PDE reduces to an ordinary differential equation (ODE).

10. **Write the compatibility condition for a system of first-order PDEs.**

    For a compatible system:

    $$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}.$$

11. **Define a general integral of a first-order PDE.**

    A **general integral** is obtained by combining complete integrals through an arbitrary relation $f(a, b) = 0$.

12. **What is a singular solution of a PDE?**

    A **singular solution** is a specific solution that cannot be obtained from the general or complete integral by choosing particular values of constants.

13. **State Cauchy's method of characteristics.**

It converts a first-order PDE $ap + bq = c$ into a set of ODEs (characteristic system):

$$\frac{dx}{a} = \frac{dy}{b} = \frac{dz}{c}.$$

Integrating these gives the solution surface.

14. **What is meant by a compatible system of first-order equations?**

A system of equations is **compatible** if it possesses at least one common solution satisfying all the equations simultaneously.

15. **Write the canonical form of a linear first-order PDE.**

$$\frac{\partial z}{\partial x} + P(x,y)\frac{\partial z}{\partial y} = Q(x,y,z).$$

16. **Explain the term "solution satisfying given condition".**

It refers to a particular solution obtained by applying specific boundary or initial conditions to a general solution.

17. **What is Jacobi's method?**

Jacobi's method is used to solve first-order PDEs of the form $F(x,y,z,p,q) = 0$ by finding relations between x, y, z, p, q satisfying:

$$\frac{dx}{F_p} = \frac{dy}{F_q} = \frac{dz}{pF_p + qF_q} = \frac{dp}{-F_x - pF_z} = \frac{dq}{-F_y - qF_z}.$$

18. **State the condition for a PDE to be exact.**

A PDE is exact if

$$\frac{\partial F}{\partial y} = \frac{\partial G}{\partial x},$$

for a differential form $F\,dx + G\,dy = 0$.

19. **What are the types of solutions of a first-order PDE?**

They are:

- Complete integral
- General integral
- Singular integral

20. **Write the form of a nonlinear first-order PDE.**

A nonlinear PDE is expressed as

$$F(x,y,z,p,q) = 0,$$

where F is a nonlinear function of p and q.

21. **What are p and q commonly used to represent in partial differential equations?**
Sol: In PDE notation for a function $z = z(x,y)$ one usually sets

$$p = \frac{\partial z}{\partial x}, \qquad q = \frac{\partial z}{\partial y}$$

22. **Write the general form of a second-order partial differential equation in two variables.**
The general second-order PDE for $u(x,y)$ is

$$A(x,y)\,u_{xx} + 2B(x,y)\,u_{xy} + C(x,y)\,u_{yy} + D\big(x,y,u,u_x,u_y\big) = 0,$$

where A, B, C and D are given functions.

23. **State the difference between the wave equation and diffusion equation.**

- *Wave equation:* second-order in time, e.g. $u_{tt} = c^2\nabla^2 u$; hyperbolic; finite propagation speed; preserves wave fronts and oscillations.
- *Diffusion equation:* first-order in time, e.g. $u_t = D\nabla^2 u$; parabolic; smoothing effect and infinite propagation speed; models dissipative spreading.

24. **What is meant by a second-order partial differential equation?**

A **second-order PDE** is an equation involving second-order partial derivatives of an unknown function of two or more independent variables. Example:

$$A\,u_{xx} + B\,u_{xy} + C\,u_{yy} + D\,u_x + E\,u_y + F\,u = G.$$

25. **What is the origin of a second-order PDE?**

A second-order PDE can be formed by eliminating two arbitrary constants from a given relation between x, y, z.

26. **When is a second-order PDE said to have constant coefficients?**

If the coefficients A, B, C, D, E, F are constants (independent of x, y), then the PDE has constant coefficients.

27. **Write the canonical form of a second-order PDE.**

$$R\,r + S\,s + T\,t + U\,p + V\,q + W\,z = 0,$$

where $r = u_{xx}, s = u_{xy}, t = u_{yy}, p = u_x, q = u_y$.

28. **How is a second-order PDE classified?**

A second-order PDE is classified based on the discriminant:

$$B^2 - 4AC$$

- > 0: Hyperbolic
- $= 0$: Parabolic
- < 0: Elliptic

29. **Write the auxiliary equation for a linear PDE with constant coefficients.**

For $F(D, D')\,z = 0$, the auxiliary equation is obtained by replacing $D = m,\ D' = 1$, so $F(m, 1) = 0$.

30. **What is the general solution of a linear PDE with constant coefficients?**

The general solution is

$$z = z_{\text{CF}} + z_{\text{PI}},$$

where z_{CF} is the complementary function and z_{PI} is the particular integral.

31. **State the rule for finding the complementary function (C.F.).**

The complementary function is obtained by solving the auxiliary equation and combining arbitrary functions corresponding to its roots.

32. **How is the particular integral (P.I.) found?**

For $F(D, D')z = f(x, y)$,

$$z_{\text{PI}} = \frac{1}{F(D, D')} f(x, y),$$

provided $F(D, D') \neq 0$ for the given $f(x, y)$.

33. **Give an example of a linear PDE with variable coefficients.**

Example:

$$x^2 u_{xx} + y^2 u_{yy} = 0.$$

34. **State the method of separation of variables.**

It assumes that the solution can be written as a product of single-variable functions:

$$u(x, y) = X(x)Y(y).$$

35. **Write the separated form of the two-dimensional Laplace equation.**
For $u_{xx} + u_{yy} = 0$, assume $u = X(x)Y(y)$, giving

$$\frac{X''}{X} + \frac{Y''}{Y} = 0.$$

Let each side equal $-\lambda$:

$$X'' + \lambda X = 0, \quad Y'' - \lambda Y = 0.$$

36. **State one application of the separation of variables method.**
Used to solve problems in heat conduction, wave motion, and Laplace's equation under given boundary conditions.

37. **What is a nonlinear second-order PDE?**
A PDE is nonlinear if the dependent variable or its derivatives appear in a nonlinear manner, e.g.,

$$(u_{xx})^2 + (u_{yy})^2 = 1.$$

38. **Give an example of a nonlinear PDE and mention its application.**
Example: Monge–Ampère equation

$$u_{xx}u_{yy} - (u_{xy})^2 = 1.$$

It arises in differential geometry and minimal surface problems.

39. **State the Laplace equation in two and three dimensions.**

$$\nabla^2 u = 0$$

In two dimensions: $u_{xx} + u_{yy} = 0$;
In three dimensions: $u_{xx} + u_{yy} + u_{zz} = 0$.

40. **What is an elementary solution of Laplace's equation?**
An **elementary solution** is a fundamental solution representing the potential at a point due to a unit source, satisfying Laplace's equation everywhere except at the origin.

41. **Write the elementary solution of Laplace's equation in three dimensions.**

$$\phi(r) = \frac{1}{4\pi r}, \quad r = \sqrt{x^2 + y^2 + z^2}.$$

42. **Write the elementary solution in two dimensions.**

$$\phi(r) = \frac{1}{2\pi} \ln r.$$

43. **What is meant by equipotential surfaces?**
Equipotential surfaces are surfaces on which the potential function $u(x, y, z)$ is constant, i.e., $u(x, y, z) = \text{constant}$.

44. **State the relationship between equipotential surfaces and field lines.**
Equipotential surfaces are always **perpendicular** to the field lines of the potential function.

45. **What is a boundary value problem (BVP)?**
A **BVP** seeks a solution of a PDE subject to specified values (or derivatives) on the boundary of the domain.

46. **Write two common types of boundary conditions.**

- **Dirichlet condition:** $u = f$ on the boundary.

- **Neumann condition:** $\dfrac{\partial u}{\partial n} = g$ on the boundary.

47. **State the Laplace equation in polar coordinates.**

$$\nabla^2 u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0.$$

48. **What is the principle of superposition in Laplace's equation?**
    If u_1 and u_2 satisfy Laplace's equation, then any linear combination $c_1u_1 + c_2u_2$ is also a solution.

49. **State the method of separation of variables for Laplace's equation.**
    Assume $u(x, y) = X(x)Y(y)$, which leads to

$$\frac{X''}{X} + \frac{Y''}{Y} = 0.$$

   Each side equals a separation constant $-\lambda$.

50. **What type of functions appear in the solution using separation of variables?**
    The solutions involve trigonometric, exponential, or hyperbolic functions depending on the sign of the separation constant.

51. **State an example of a boundary value problem in a rectangular domain.**
    Find $u(x, y)$ such that

$$u_{xx} + u_{yy} = 0, \quad u(0, y) = 0,\ u(a, y) = 0,\ u(x, 0) = 0,\ u(x, b) = f(x).$$

52. **What is meant by a surface boundary value problem?**
    It is a PDE problem where the boundary conditions are specified on a surface enclosing the region, e.g., a sphere or cylinder.

53. **Write the Laplace equation in cylindrical coordinates for axial symmetry.**
    For $u = u(r, z)$,

$$u_{rr} + \frac{1}{r}u_r + u_{zz} = 0.$$

54. **What is a problem with axial symmetry?**
    A problem where the potential depends only on the distance r from the axis and z (not on the azimuthal angle θ).

55. **What is the significance of the Green's function in solving Laplace's equation?**
    The Green's function represents the potential due to a unit point source and is used to construct the solution of Laplace's equation with given boundary conditions.

56. **State one physical application of Laplace's equation.**
    Laplace's equation describes steady-state heat conduction, electrostatic potential, and incompressible fluid flow.

57. **Write the general form of the one-dimensional diffusion equation.**

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$$

   where D is the diffusion coefficient.

58. **State the physical meaning of the diffusion coefficient D.**
    It represents the rate at which a substance diffuses and has units of m^2/s.

59. **What is the elementary solution of the one-dimensional diffusion equation?**

$$u(x, t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right)$$

60. **Explain why the diffusion equation is parabolic in nature.**
Because it involves a first-order time derivative and a second-order spatial derivative, indicating diffusive (non-wave) propagation.

61. **Write the initial condition for the elementary diffusion solution.**

$$u(x, 0) = \delta(x)$$

where $\delta(x)$ is the Dirac delta function.

62. **State the method of separation of variables.**
Assume $u(x, t) = X(x)T(t)$, separate variables, and solve ODEs for X and T.

63. **Write the separated equations obtained from $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$.**

$$\frac{1}{DT}\frac{dT}{dt} = \frac{1}{X}\frac{d^2 X}{dx^2} = -\lambda$$

64. **Give the general solution for $T(t)$ in the separation of variables method.**

$$T(t) = Ae^{-D\lambda t}$$

65. **What type of spatial solutions are obtained from separation of variables for the diffusion equation?**
Sinusoidal or exponential functions depending on boundary conditions.

66. **What boundary conditions are often applied to diffusion problems?**
Typically, $u(0, t) = u(L, t) = 0$ or $\frac{\partial u}{\partial x} = 0$ on boundaries.

67. **State one property of the Green's function for the diffusion equation.**
It satisfies the equation:

$$\frac{\partial G}{\partial t} - D\frac{\partial^2 G}{\partial x^2} = \delta(x - \xi)\delta(t - \tau)$$

68. **What is the role of Green's function in solving the diffusion equation?**
It expresses the solution as a convolution of the Green's function with initial and boundary data.

69. **Write the general integral solution using the Green's function.**

$$u(x, t) = \int_{-\infty}^{\infty} G(x, \xi, t) f(\xi)\, d\xi$$

70. **What is the Green's function for the 1D infinite domain diffusion equation?**

$$G(x, \xi, t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{(x - \xi)^2}{4Dt}\right)$$

71. **Explain the physical meaning of the Green's function for diffusion.**
It represents the temperature (or concentration) response at point x due to an instantaneous point source at ξ.

72. **State one important property of the diffusion Green's function regarding normalization.**

$$\int_{-\infty}^{\infty} G(x, \xi, t)\, dx = 1$$

This ensures conservation of total mass (or energy).