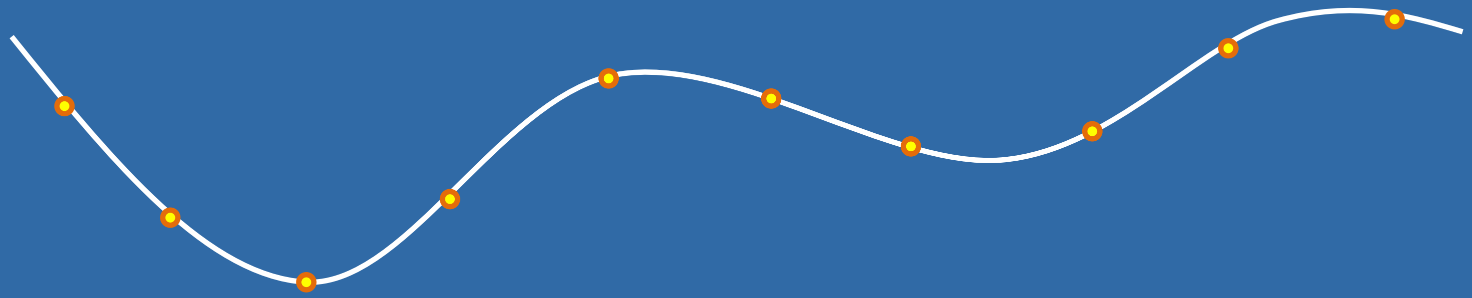
FLUID DYNAMICS

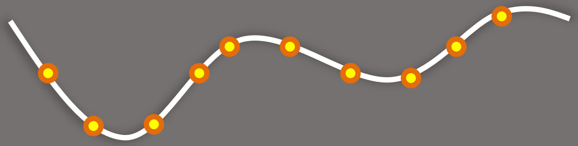
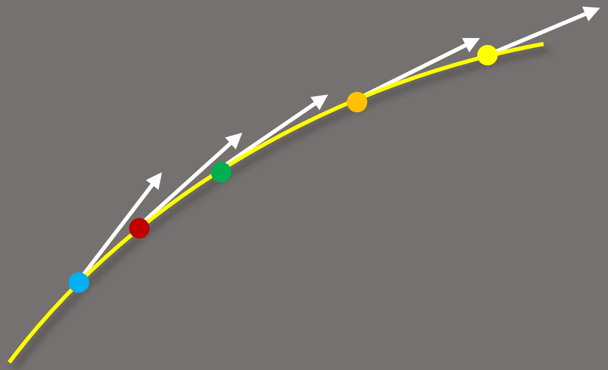
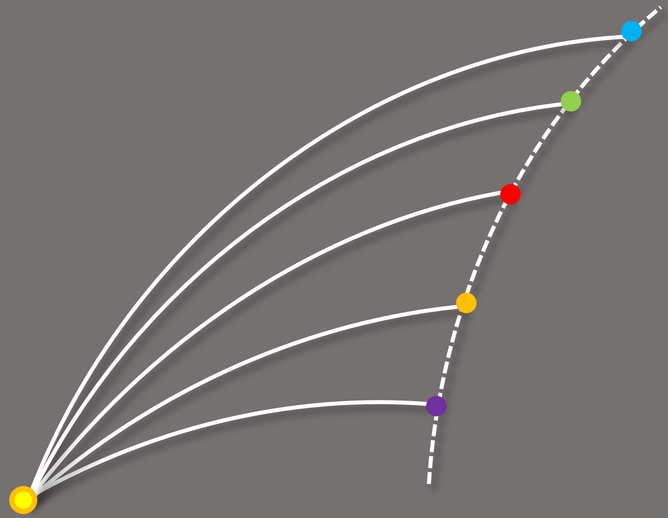
Pathline

Streamline

Streakline



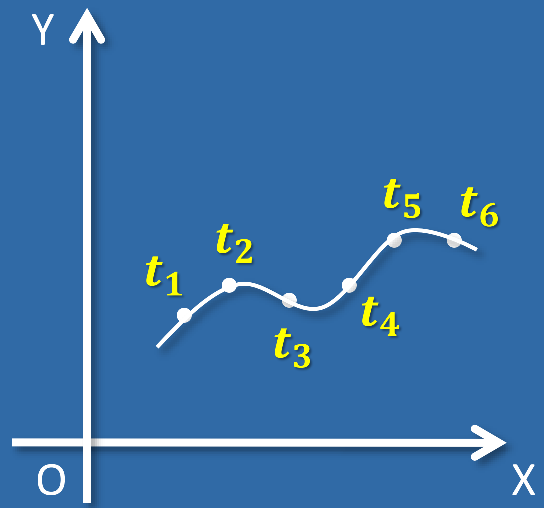
Pathline

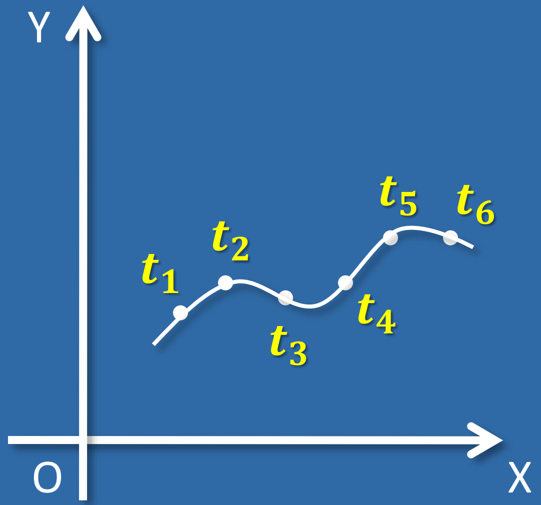
Pathline

The curve described in space by moving fluid element is known as pathline i.e. a pathline is a line traced by a particle in the fluid.

The pathline shows the direction of the velocity of the fluid particle at any instant of time. Such a line is obtained by giving the position of an element as a function of time.



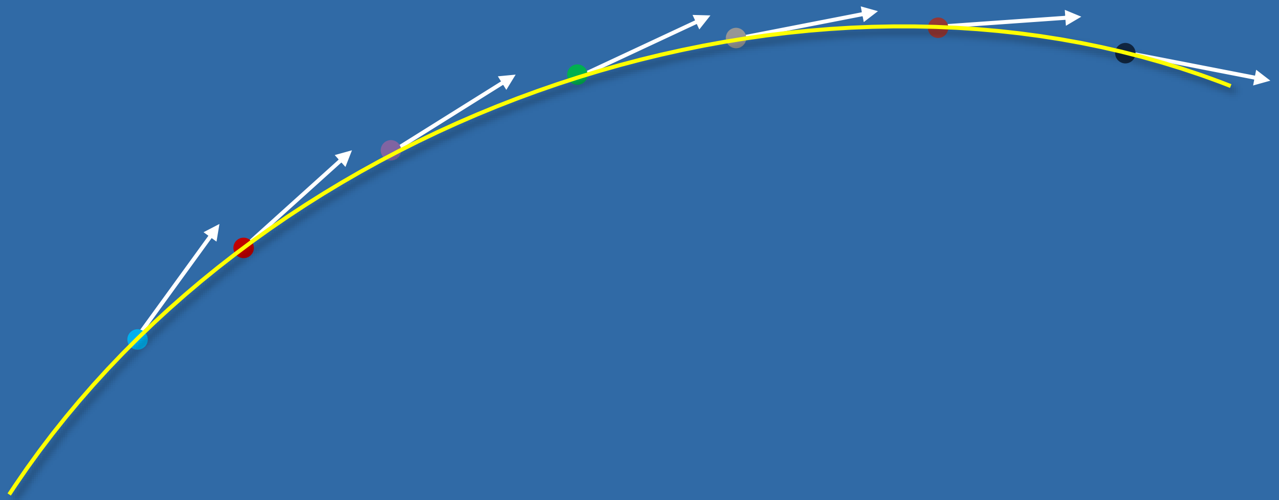
Pathline

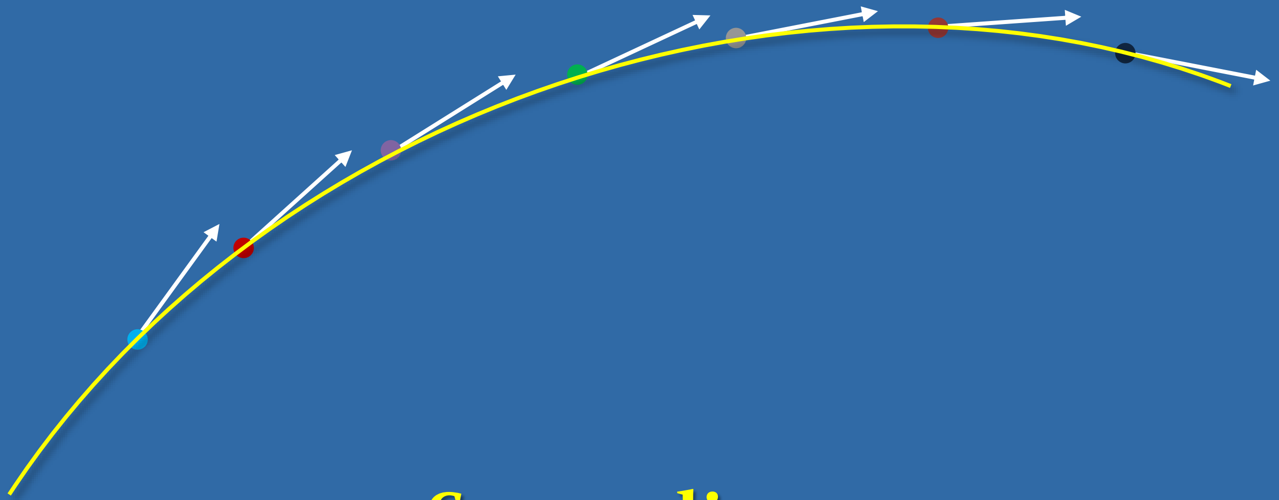


The differential equation of pathlines are $\dfrac{d\vec{r}}{dt} = \vec{q}$

$$\frac{dx}{dt} = u, \qquad \frac{dy}{dt} = v, \qquad \frac{dz}{dt} = w$$

The pathlines vary with each fluid particle. It represents the direction of velocity of a single particle of fluid at various time.





Streamline

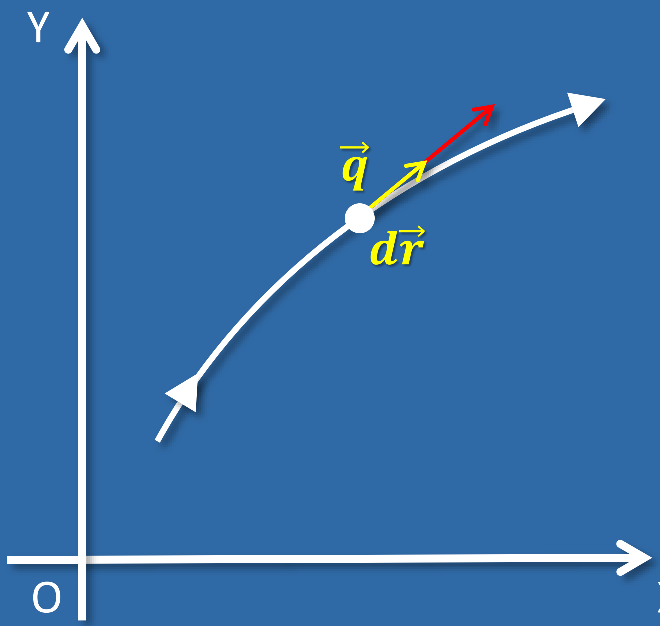
Streamline

A streamline is a continuous line of flow drawn in the fluid so that the tangent at every point of it at any instant of time coincides with the direction of motion of the fluid at that point.

Or,

It is an imaginary line in the fluid flow, tangent to which at any point represents the direction of velocity of fluid particle at that point.

Let us consider $d\vec{r}$ be an element of the streamline passing through any point $P(r)$ at an instant of time. Let $\vec{q}$ be the velocity at that point at the same instant. The direction of the tangent and direction of velocity are parallel, i.e.



$$d\vec{r} \times \vec{q} = 0$$

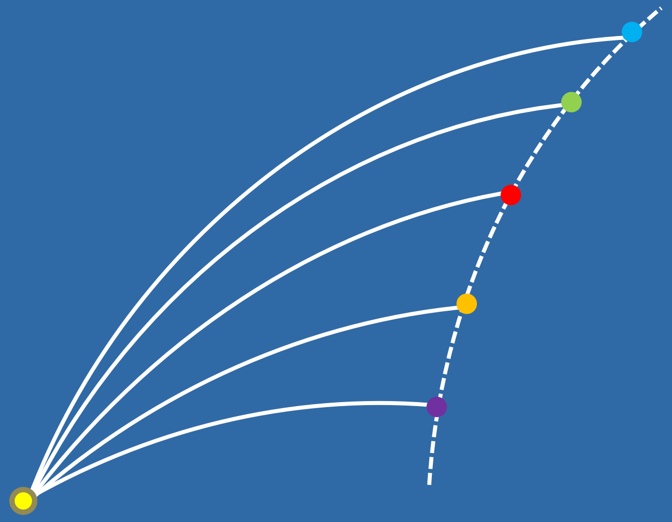
$$=> \left(dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}\right) \times \left(u\hat{\imath} + v\hat{\jmath} + w\hat{k}\right) = 0$$

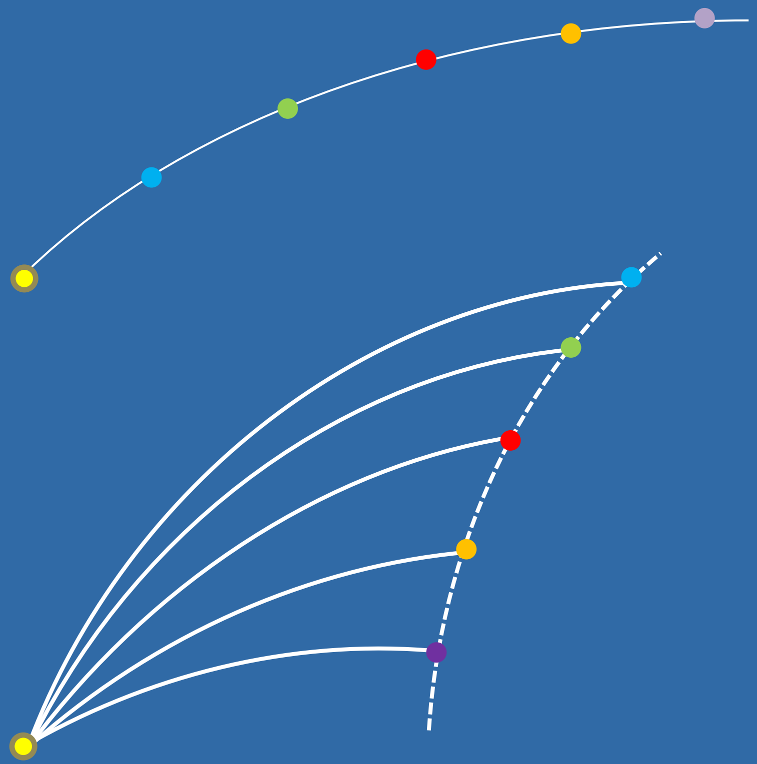
$$=> (vdz - wdy)\hat{i} + (wdx - udz)\hat{j} + (udy - vdx)\hat{k} = 0$$

$$=> \ vdz - wdy = 0, wdx - udz = 0, wdx - udz = 0$$

$$=> \ \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

which is known as the differential equation for a streamline where the velocity components u, v, w are the functions of x, y, z and t. When the flow is steady, the streamlines have the same form at all times. When the flow is unsteady the streamline change from instant to instant.

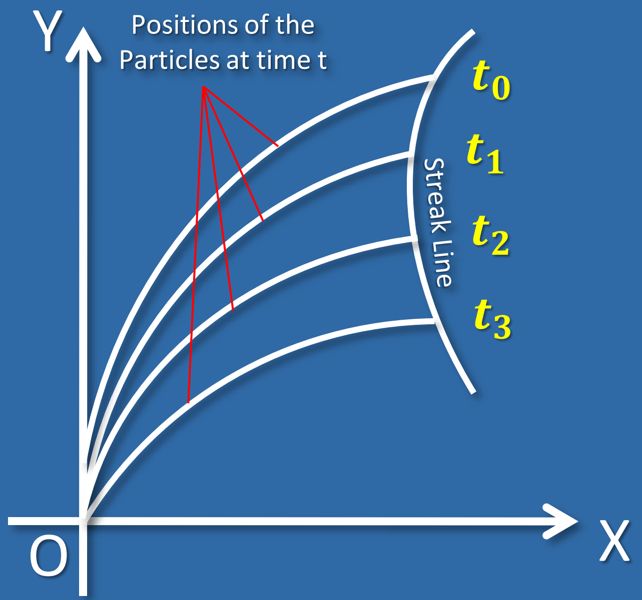




Streaklines

Streakline

A streakline is a line on which lie all those fluid elements that at some earlier instant passed through a particular point in space. It is a line making the position of a set of fluid particles that had passed through a fixed point in the flow field. A streakline is defined as the locus of different particles passing through fixed point. A streakline connects the



locations of the particles at one instant moving with the fluid which passed through a particular point.

Let us consider a fluid particle (x_0, y_0, z_0) passes a fixed point $\overrightarrow{r_1}(x_1, y_1, z_1)$ in the course of time. By Lagrangian description of fluid flow, we have

$$x_1 = f_1(x_0, y_0, z_0, t)$$
$$y_1 = f_2(x_0, y_0, z_0, t)$$
$$z_1 = f_3(x_0, y_0, z_0, t)$$

Solving the equations for x_0, y_0, z_0 we have

$$x_0 = g_1(x_1, y_1, z_1, t)$$
$$y_0 = g_2(x_1, y_1, z_1, t)$$
$$z_0 = g_3(x_1, y_1, z_1, t)$$

Since a streakline is the locus of the positions (x, y, z) of the particles which have passed through the fixed point (x_1, y_1, z_1) therefore the equation of the streakline at an instant of time t is given by

$$x = h_1(x_0, y_0, z_0, t)$$
$$y = h_2(x_0, y_0, z_0, t)$$
$$z = h_3(x_0, y_0, z_0, t)$$

Hence the streakline passing through the fixed point is

$$x = h_1(g_1, g_2, g_3, t)$$
$$y = h_2(g_1, g_2, g_3, t)$$
$$z = h_3(g_1, g_2, g_3, t)$$

NOTE: When the flow is steady the streaklines, streamlines and pathlines coincide.

Q: The velocity vector $\vec{q}$ is given by $\vec{q} = x\hat{i} - y\hat{j}$. Determine the equation of streamline.

Sol: From the definition of a streamline,

$$\vec{q} \times d\vec{r} = 0$$

$$=> (x\hat{i} - y\hat{j}) \times (\hat{i}dx + \hat{j}dy) = 0$$

$$=> (xdy + ydx)\hat{k} = 0$$

$$=> \frac{dx}{x} = -\frac{dy}{y}$$

Integrating,

$$=> logx + logy = logc$$

$$=> xy = c$$

which represents the rectangular hyperbolas where c is an arbitrary constant.

Q: The velocity $\vec{q}$ in a three dimensional flow field for an incompressible fluid is given by $\vec{q} = 2x\hat{\imath} - y\hat{\jmath} - z\hat{k}$. Determine the equations of the streamlines passing through the point (1, 1, 1).

Sol: The velocity $\vec{q}$ in a three dimensional flow field for an incompressible fluid is given by $\vec{q} = 2x\hat{\imath} - y\hat{\jmath} - z\hat{k}$.

The equations of streamlines are given by

$$\frac{dx}{u} = \frac{dy}{u} = \frac{dz}{w}$$

$$=> \frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{-z}$$

From the first two ratios

$$\frac{dx}{2x} = \frac{dy}{-y}$$

$$=> \frac{dx}{x} + \frac{2dy}{-y} = 0$$

Integrating,

$$\log x + 2\log y = \log A$$

$$=> xy^2 = A$$

From the first and third ratios

$$\frac{dx}{2x} = \frac{dz}{-z}$$

$$=> \frac{dx}{x} + \frac{2dz}{z} = 0$$

Integrating, $$\log x + 2\log z = \log B$$

$$=> xy^2 = B$$

At the point (1,1,1), $A = B = 1$

Hence the required streamlines are $xy^2 = A$, $xy^2 = B$.

Q: Determine the streamlines and pathlines of the particles

$$u = \frac{x}{1+t}, v = \frac{y}{1+t}, w = \frac{z}{1+t}$$

Sol: Given

$$u = \frac{x}{1+t}, v = \frac{y}{1+t}, w = \frac{z}{1+t}$$

The equations of streamlines are given by

$$\frac{dx}{u} = \frac{dy}{u} = \frac{dz}{w}$$

$$=> \frac{\frac{dx}{x}}{1+t} = \frac{\frac{dy}{y}}{1+t} = \frac{\frac{dz}{z}}{1+t}$$

$$=> \frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

From 1st and 2nd ratio,

$$\frac{dx}{x} = \frac{dy}{y}$$

$$=> \log x = \log y + \log A$$

$$=> x = Ay$$

From 1st and 3rd ratio,

$$\frac{dy}{y} = \frac{dz}{z}$$

$$=> \log x = \log z + \log B$$

$$=> x = Bz$$

Hence the streamlines are given by the intersection of $x = Ay$ and $x = Bz$

The differential equation of pathlines is given by

$$\vec{q} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$

$$=> \frac{x}{1+t}\hat{\imath} + \frac{y}{1+t}\hat{\jmath} + \frac{z}{1+t}\hat{k} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$

$$\frac{dx}{dt} = \frac{x}{1+t}$$

$$\frac{dy}{dt} = \frac{y}{1+t}$$

$$\frac{dz}{dt} = \frac{z}{1+t}$$

Now,

$$\frac{dx}{dt} = \frac{x}{1+t}$$

$$=> \frac{dx}{x} = \frac{dt}{1+t}$$

Integrating,

$$x = A(1+t)$$

Similarly,

$$y = B(1+t)$$

$$z = C(1+t)$$

which gives the required path of the particles.

THANK YOU