SPECIAL THEORY OF RELATIVITY

MAT 305

VARIATION OF MASS WITH VELOCITY

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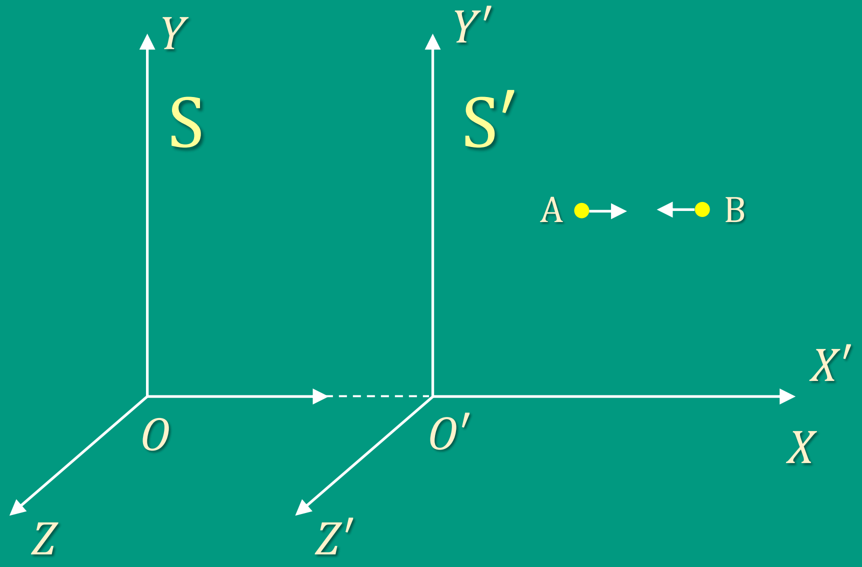
Mass is a function of the velocity of the body. It increases with increasing velocity represented by the relation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m_0 is the mass of the object at rest, m is the mass of the object moving with velocity v, c is the velocity of light, v is the velocity of the object.

Let us consider two frame of references S and S' where S' is moving away from S with a constant velocity v relative to S.



Let u and $-u$ be the velocity of the balls A and B w.r.t. S' frame.

Let u_1 and u_2 be the velocity of the balls A nd B w.r.t. frame of reference S.

$$u_1 = \frac{u+v}{1+\frac{uv}{c^2}} \qquad (1)$$

$$u_2 = \frac{-u+v}{1-\frac{uv}{c^2}} \qquad (2)$$

Applying law of conservation of momentum for frame of reference *S*.

Total initial momentum before collision = Total final momentum after collision

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$=> m_1\left(\frac{u+v}{1+\frac{uv}{c^2}}\right) + m_2\left(\frac{-u+v}{1-\frac{uv}{c^2}}\right) = (m_1 + m_2)v$$

$$=> m_1\left(\frac{u+v}{1+\frac{uv}{c^2}}\right) - m_1v = m_2v - m_2\left(\frac{-u+v}{1-\frac{uv}{c^2}}\right)$$

$$=> m_1\left(\frac{u-u\frac{v^2}{c^2}}{1+\frac{uv}{c^2}}\right) = m_2\left(\frac{u-u\frac{v^2}{c^2}}{1-\frac{uv}{c^2}}\right)$$

$$=> \frac{m_1}{m_2} = \frac{1 + \frac{uv}{c^2}}{1 - \frac{uv}{c^2}} \qquad (3)$$

$$(1)^2 => \quad {u_1}^2 = \left(\frac{u + v}{1 + \frac{uv}{c^2}} \right)^2$$

$$=> 1 - \frac{{u_1}^2}{c^2} = 1 - \frac{1}{c^2} \left(\frac{u + v}{1 + \frac{uv}{c^2}} \right)^2$$

$$=> 1 - \frac{{u_1}^2}{c^2} = \frac{\left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{v^2}{c^2}\right)}{\left(1 + \frac{uv}{c^2}\right)^2} \qquad (4)$$

Similarly,

$$1 - \frac{{u_2}^2}{c^2} = \frac{\left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{v^2}{c^2}\right)}{\left(1 - \frac{uv}{c^2}\right)^2} \qquad (5)$$

Dividing (5) by (4),

$$\frac{1-\frac{{u_2}^2}{c^2}}{1-\frac{{u_1}^2}{c^2}}=\frac{\left(1+\frac{uv}{c^2}\right)^2}{\left(1-\frac{uv}{c^2}\right)^2}$$

$$=>\frac{1+\frac{uv}{c^2}}{1-\frac{uv}{c^2}}=\frac{\sqrt{1-\frac{{u_2}^2}{c^2}}}{\sqrt{1-\frac{{u_1}^2}{c^2}}}$$

$$=>\frac{m_1}{m_2}=\frac{\sqrt{1-\frac{{u_2}^2}{c^2}}}{\sqrt{1-\frac{{u_1}^2}{c^2}}}$$

$$=> m_1\sqrt{1-\frac{{u_1}^2}{c^2}} = m_2\sqrt{1-\frac{{u_2}^2}{c^2}} = m_0 \quad \text{(any constant)}$$

$$=> m_1\sqrt{1-\frac{{u_1}^2}{c^2}} = m_0$$

$$=> m_1 = \frac{m_0}{\sqrt{1-\frac{{u_1}^2}{c^2}}}$$

Let, $m_1 = m, u_1 = v$

$$\therefore \quad m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \blacksquare$$