SPECIAL THEORY OF RELATIVITY

MAT 305

EQUIVALENCE OF MASS AND ENERGY

MASS-ENERGY RELATION

The mass 'm' of a body moving with velocity 'v' relative to a stationary observer varies with 'v' and is given by-

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1)$$

where m_0 is the mass of the object at rest, m is the mass of the object moving with velocity v, c is the velocity of light, v is the velocity of the object.

Squaring both sides of equation (1) we get,

$$m_0^2 = m^2 \left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow m^2c^2 - m^2v^2 = m_0^2c^2$$

On differentiation we get,

$$\Rightarrow 2mc^2dm - [2mv^2dm + 2m^2vdv] = 0$$

$$\Rightarrow c^2dm = v^2dm + mvdv \qquad (2)$$

Now,

$$\text{Workdone} \;= dk = dw = Fds \qquad (3)$$

From Newton's 2nd law of motion,

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

From equation (3) we get,

$$dk = \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right)ds$$

$$=> dk = m\frac{ds}{dt}dv + v\frac{ds}{dt}dm$$

$$=> dk = mvdv + v^2dm \qquad (4)$$

Comparing equations (2) and (4) we get,

$$dk = c^2dm$$

Integrating we get,

$$\int_0^k dk = \int_{m_0}^m c^2dm$$

$$=> k = c^2(m - m_0)$$

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which is relativistic kinetic energy.

Rest mass energy = $m_0 c^2$

Total Energy = Kinetic Energy + Rest Mass Energy

$$= c^2(m - m_0) + m_0 c^2$$

$$= mc^2$$

$$\therefore \quad E = mc^2$$